Two-Way Coherent Doppler Error Due to Solar Corona

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This report considers two-way coherent Doppler error resulting from phase scintillations induced on the uplink by the solar corona. It is shown that this error can be estimated by taking statistics on the differential Doppler measurements. Typical estimates for the error are given for four Sun-Earth-probe angles and for integration times ranging from 1 second to 1 minute. These results are based on data collected during the 1985 Voyager 2 solar conjunction.

I. Introduction

When a spacecraft is in the same area of the sky as the Sun and beyond it, the radio beams to and from that spacecraft experience phase scintillations due to passage through the solar corona. These scintillations will dominate all other sources of error for two-way coherent Doppler measurement when the Sun-Earth-probe angle is small. Thus, for those missions that feature numerous or long-lasting solar conjunctions, especially inner planetary missions, it is important to characterize this source of Doppler error. Unfortunately, the relevant parameters of the solar corona are highly variable, and adequate statistical characterizations do not exist. It has, however, been possible to measure the phase scintillations induced on 2.3- and 8.4-GHz downlinks by the solar corona. It is in fact possible to make this measurement using any Sun-encountering spacecraft with two downlinks of different but related frequencies. The phase scintillations induced on the downlinks, once measured, can be removed from the Doppler phase record. The phase scintillations induced on the uplink, on the other hand, remain. These cannot be measured or removed unless two simultaneous uplinks are available.

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II. Measuring Phase Scintillations Induced on the Downlinks

Whenever a two-way coherent Doppler measurement is being performed, the downlink frequencies f_1 and f_2 are related to the uplink frequency f_0 by the transponding ratios G_1 and G_2 and the relative velocity of the spacecraft and the deep space station. In addition, there are noise terms that represent the phase scintillations picked up in transit through the solar corona. These frequency relationships may be expressed by

$$f_i = G_i f_0 (1 - 2\dot{\rho}/c) + \frac{G_i v_u^2}{f_0} + \frac{v_d^2}{f_0 G_i} ; \qquad i = 1, 2 \quad (1)$$

The nonrelativistic Doppler shift has been indicated here; it is proportional to the ratio of the range rate $\dot{\rho}$ to the speed of light, c.

The parameter ν_u^2, in units of Hz^2, represents the level of phase scintillations induced on the uplink by the solar corona. It depends only on the physical properties of that part of the corona through which the uplink radio beam passes. It is an implicit function of time. By taking the integral of the square of the local plasma frequency along the uplink ray path, then taking a time derivative and dividing by twice the speed of light, one could calculate ν_u^2. Unfortunately, the local plasma frequency is generally not known. The parameter ν_d^2 is the downlink analog of ν_u^2.

Using Eq. (1), we can obtain an expression for ν_d^2:

$$\nu_d^2 = G_1 f_0 \left[1 - (G_1/G_2)^2 \right]^{-1} [f_1 - (G_1/G_2)f_2] \qquad (2)$$

The phase scintillations induced on the downlinks contribute a frequency noise term that can be identified if the differential Doppler shift, $f_1 - (G_1/G_2)f_2$, is measured. Once identified, this frequency noise term is easily subtracted out of the Doppler record. At this point, the only error due to solar corona remaining in the Doppler record is the term involving ν_u^2. The Doppler error ϵ, in velocity units, is

$$\epsilon = \frac{c}{2 f_0^2} \langle \nu_u^2 \rangle_T \qquad (3)$$

The brackets $\langle \cdot \rangle_T$ indicate a time average over an integration time T.

The statistics of ν_u^2 and ν_d^2 are the same. This is in fact the key to being able to estimate the error ϵ. In the writing of standard deviations $\sigma(\cdot)$, then, the subscripts u and d may be ignored.

$$\sigma(\epsilon) = \frac{c}{2 f_0^2} \sigma(\langle \nu^2 \rangle_T) \qquad (4)$$

As suggested by Eq. (2), the required coronal statistics can be obtained by taking a statistical measure of the differential Doppler shift, viz.,

$$\sigma(\langle \nu^2 \rangle_T) = G_1 f_0 \left[1 - (G_1/G_2)^2 \right]^{-1} \sigma(\langle f_1 - (G_1/G_2)f_2 \rangle_T)$$
$$\qquad (5)$$

The required coronal statistics can also be obtained from a pair of one-way downlinks of different but related frequencies. This is, in fact, how the data appearing in this report were obtained. In this case, Eqs. (4) and (5) still hold, but some terms in Eq. (5) need proper interpretation. The term $G_1 f_0$ is the frequency of the first downlink, and the term G_1/G_2 is the ratio of the downlink frequencies.

III. Voyager 2 Results

During the 1985 solar conjunction for Voyager 2, differential Doppler data were collected from a pair of one-way downlinks. The downlinks originated with an ultrastable oscillator aboard the spacecraft, and the 64-m subnet was used for reception. Estimates were made of what the observed level of phase scintillations would do to a two-way coherent Doppler measurement. These estimates were obtained by applying the differential Doppler statistics to Eqs. (4) and (5). The frequency of the first downlink was approximately 2296 MHz. The ratio of the downlink frequencies, G_1/G_2, was 3/11.

The standard deviation of the two-way coherent Doppler error, as expressed in Eq. (4) and based on Voyager 2 differential Doppler measurements, has been compiled in Table 1 and plotted in Fig. 1. The statistics have been calculated for integration times ranging from 1 second to 1 minute. For the longer integration times, the error is less. Four example Sun-Earth-probe angles have been included. It must be understood that the properties of the solar corona are highly variable and that the results shown in Table 1 and Fig. 1 do not represent a characterization of the error as a function of Sun-Earth-probe angle. Such a characterization must be based on a larger set of data.

The Doppler error due to solar corona can be reduced by using a higher frequency on the uplink. For 7.2-GHz and 34.3-GHz uplinks, the errors shown in Table 1 and Fig. 1 are divided by 11.5 and 263, respectively. For some missions, this is an important advantage for the higher frequencies.

IV. Conclusions

This report has explained how phase scintillations induced on the downlinks can be measured. It has been shown that taking statistics on these measured phase scintillations leads to estimates of the Doppler error due to uplink phase scintillations. Typical estimates for the error have been calculated based on differential Doppler data collected during the 1985 Voyager 2 solar conjunction.

Table 1. Two-way coherent Doppler error for 2.1-GHz uplink and dual-frequency calibrated downlink*

Integration time, sec	Doppler error (mm/sec) for several Sun-Earth-probe angles			
	1.3°	1.7°	2.1°	2.5°
1	172.0	137.2	99.4	41.3
5	161.7	98.2	65.7	31.2
10	152.6	78.1	54.2	27.2
15	142.7	67.1	47.9	24.3
20	137.1	64.7	45.3	23.2
25	131.7	57.9	40.2	22.8
30	127.7	53.5	41.1	21.7
35	123.9	52.9	37.9	21.3
40	122.3	51.5	37.2	19.4
45	117.7	48.7	33.9	19.6
50	117.9	47.6	34.0	20.5
55	116.5	45.9	34.1	19.2
60	112.4	43.5	33.4	18.4

*These results are based on phase scintillation measurements made using the Voyager 2 spacecraft during its 1985 solar conjunction. The time and place at which the measurements were made are indicated below.

1.3°: 2030 to 2135 on 12-8-85 at Goldstone
1.7°: 0440 to 0640 on 12-12-85 at Canberra
2.1°: 0435 to 0545 on 12-8-85 at Canberra
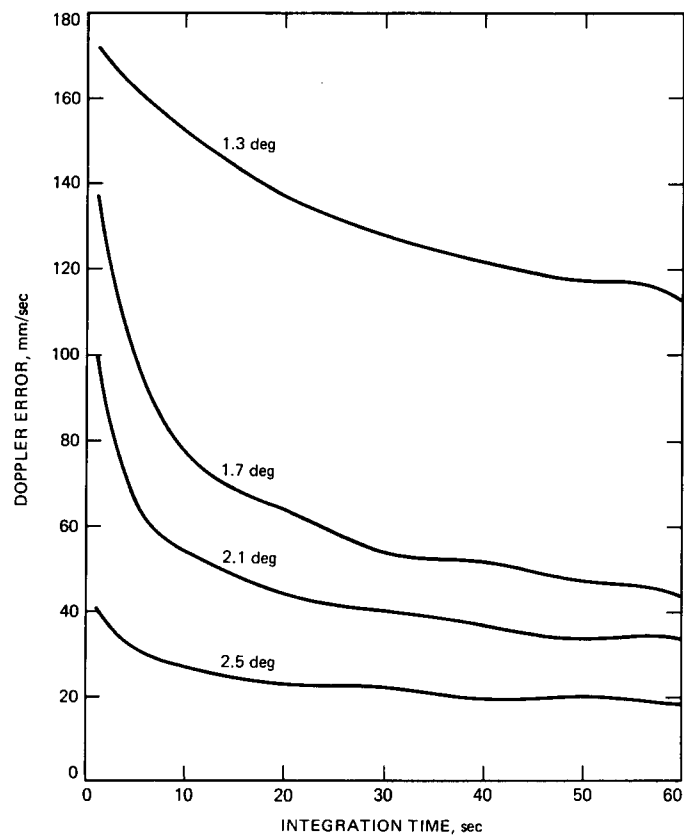2.5°: 2015 to 2120 on 12-12-85 at Goldstone

**Fig. 1. Doppler error due to solar corona for various
Sun-Earth-probe angles.**